ON SPACE TRIANGULATION ADJUSTMENT USING MEASURED LENGTH AND ANGLE COORDINATE VALUES

by

J. Lorinczi/ Roumania/

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Some aspects of the condition equations for the space triangulation network are presented in this paper, by using the results of the simultaneous measurements of the topocentrical coordinates and that of the distances from the tracking stations to the satellite. The mathematical model for the network adjustment is given by system /7/ by applying the simultaneity circle method. The most probable values for the measured quantities/ the distances and the angular coordinates/ and the unmeasured ones/ the lengths and directions of the chords linking the satellite tracking stations/ are obtained from the adjustment.

I. Let us assume that A and B are two earth stations tracking a satellite P is the celestial pole; S is the satellite position at a given moment; $S^A/\alpha A, \delta^A/$ and $S^B/\alpha B, \delta^B/$ - is the projection in a celestial sphere of the satellite position S from A and B and their corresponding equatorial topocentric coordinates at the moment of observation; γ/A,D,/ - is the projection on a celestial sphere the range or extent of the direction connecting stations A and B; r^A and r^B are the distances from A and B to S at the moment of observation. Let us designate by letters a,b,c, the interior and exterior angles of triangles of ABS, where $a=\widehat{SAQ}$, $b=\widehat{SBQ}$ and $C=\widehat{ASB}$.

The angular coordinates from S^A, S^B and Q, corresponding to the given position of the satellite, are connected with the adjustment of the simultaneity circle method [1]. The linear position of this conditional adjustment is given in article [2]. The length of the sides corresponding to this satellite position is connected with angular coordinates with one of the following equations:

$$\sqrt{\rho^{A^2}+\rho^{B^2}-2\rho^A\rho^B\cos c} \qquad - L = 0, \text{ if } r^A \text{ and } r^B \text{ are measured;}$$

$$\rho^A\sqrt{\frac{1-\cos^2 c}{1-\cos^2 b}} \qquad - L = 0, \text{ only if } r^A \text{ is measured;}$$

$$\rho^B\sqrt{\frac{1-\cos^2 c}{1-\cos^2 a}} \qquad - L = 0, \text{ only if } r^B \text{ is measured;}$$

where L - length of the sides AB, while

$$\left.\begin{aligned} \cos a &= \sin\delta^A \sin D + \cos\delta^A \cos D \cos(\alpha^A - A) \\ \cos b &= \sin\delta^B \sin D + \cos\delta^B \cos D \cos(\alpha^B - A) \\ \cos c &= \sin\delta^A \sin\delta^B + \cos\delta^A \cos\delta^B \cos(\alpha^A - \alpha^B) \end{aligned}\right\} \quad /1/$$

"The linearly angular" condition of the equation will have the following expression:

$$b_1' v_{\rho^A} + b_2' v_{\rho^B} + a_1' v_{\alpha^A} + a_2' v_{\delta^A} + a_3' v_{\alpha^B} + a_4' v_{\delta^B} - \Delta L + \alpha' \Delta A + \beta' \Delta D + w_\alpha' = 0, \qquad /2/$$

where v_{ρ^A}, v_{ρ^B}, v_{α^A}, v_{δ^A}, v_{α^B}, v_{δ^B} -are corrections in the measured quantities, a ΔL, ΔA, ΔD are the corrections in the non-measured quantities. A coefficience in free members are expressed by the following formulas:

<u>Case I</u>. Measured ρ^A и ρ^B

$$\begin{aligned} b_1' &= \frac{\rho_0^A - \rho_0^B \cos(c)}{(L)}; & b_2' &= \frac{\rho_0^B - \rho_0^A \cos(c)}{(L)}; \\ a_1' &= k\, g(A,B); & a_2' &= -k\, f(A,B); \\ a_3' &= k\, g(B,A); & a_4' &= -k\, f(B,A); \qquad /3/ \\ \alpha' &= 0; \quad \beta' = 0 & w_\alpha' &= (L) - \overline{L} \\ k &= \frac{\rho_0^A \rho_0^B}{\rho'(L)} & (L) &= \sqrt{\rho_0^{A^2} + \rho_0^{B^2} - 2\rho_0^A \rho_0^B \cos(c)} \end{aligned}$$

L = the approximated values of the length of sides AB.

Case II. Only r_o^A is measured.

$$
\begin{aligned}
&b_1' = \frac{\sin(c)}{\sin(b)}; && b_2' = 0; \\
&a_1' = m_A g(A,B); && a_2' = -m_A f(A,B); \\
&a_3' = m_A g(B,A) - n_A g(B,Q); && a_4' = -m_A f(B,A) + n_A f(B,Q);
\end{aligned}
\qquad /4/
$$

$$
\alpha' = n_A g(B,Q); \quad \beta' = n_A f(B,Q); \quad w_a' = (L) - \bar{L};
$$

$$
(L) = r_o^A \frac{\sin(c)}{\sin(b)}; \quad m_A = \frac{r_o^A}{\rho''} \cdot \frac{\operatorname{ctg}(c)}{\sin(b)}; \quad n_A = \frac{r_o^A}{\rho''} \cdot \frac{\sin(c)\cos(b)}{\sin^2(b)}
$$

L - the approximated values of the length of sides AB.

Case III. Only r^B is measured.

$$
\begin{aligned}
&b_1' = 0; && b_2' = \frac{\sin(c)}{\sin(a)}; \\
&a_1' = m_B g(A,B) - n_B g(A,Q); && a_2' = -m_B f(A,B) + n_B f(A,Q); \\
&a_3' = m_B g(B,A); && a_4' = -m_B f(B,A);
\end{aligned}
\qquad /5/
$$

$$
\alpha' = n_B g(A,Q); \quad \beta' = n_B f(A,Q); \quad w_a' = (L) - \bar{L};
$$

$$
(L) = r_o^B \frac{\sin(c)}{\sin(a)}; \quad m_B = \frac{r_o^B}{\rho''} \cdot \frac{\operatorname{ctg}(c)}{\sin(a)}; \quad n_B = \frac{r_o^B}{\rho''} \cdot \frac{\sin(c)\cos(a)}{\sin^2(a)}.
$$

L - the approximated values of the length of the sides AB.

In all three cases we obtain:

1) $r_o^A,\ r_o^B,\ \alpha_o^A,\ \delta_o^A,\ \alpha_o^B,\ \delta_o^B$ - the measured quantities;

2) $(a),\ (b),\ (c),\ (L),\ \bar{L}$ - the non-measured quantities;

$$
\begin{aligned}
&f(i,k) = \cos\delta_o^i \sin\delta_o^k - \sin\delta_o^i \cos\delta_o^k \cos(\alpha_o^i - \alpha_o^k); \\
&g(i,k) = \cos\delta_o^i \cos\delta_o^k \sin(\alpha_o^i - \alpha_o^k); \\
&i = A, B; \quad k = A, B, Q; \quad (i \neq k); \quad \rho'' = 206.264{,}8''; \\
&f(i,k) \neq f(k,i); \quad g(i,k) = -g(k,i)
\end{aligned}
\qquad /6/
$$

2. Besides the two above mentioned forms of conditional equations of "angular" and accordingly "linearly angular" conditional equations for trangles, formed from chords are constructed by one angular conditional equations relative to the unmeasured quantities ΔA and ΔD [2].

3. We now assume a cosmic triangular network having a chord t. At the ends of each chord, are effected simultaneous measurements of angular cooridnates and the length of the sides. For each observed position of the satellite there corresponds one "angular" [2] and one "linear angular" conditional adjustment of the form [2]. The system of conditional adjustments of the entire network in a matrix presentation is as follows:

$$\begin{cases} D_i'^{*} V_i' + A_i'^{*} V_i + B_i' X_i' + W_i' = 0 \\ A_i^{*} V_i + B_i X_i + W = 0 \\ (i = 1, 2, \ldots, t) \\ C_i^{*} X_i + \overline{W} = 0 \;{}^{1)} \end{cases} \qquad /7/$$

where

$$D_i' = \begin{Vmatrix} D_{i1}' & & & \\ & D_{i2}' & & \\ & & \ddots & \\ & & & D_{in_i}' \end{Vmatrix}; \quad D_{ik}' = \begin{Vmatrix} b_{1ik}' \\ \\ b_{2ik}' \end{Vmatrix}; \quad V_i' = \begin{Vmatrix} V_{i1}' \\ V_{i2}' \\ \vdots \\ V_{in_i}' \end{Vmatrix}; \quad V_{ik}' = \begin{Vmatrix} v_{x\,ik}^{A} \\ \\ v_{x\,ik}^{B} \end{Vmatrix}; \qquad /7/a$$

I/ *- Transposition Sign

$$A'_i = \begin{Vmatrix} A'_{i1} & & & \\ & A'_{i2} & & \\ & & \ddots & \\ & & & A'_{in_i} \end{Vmatrix}; \quad A'_{ik} = \begin{Vmatrix} a'_{1ik} \\ a'_{2ik} \\ a'_{3ik} \\ a'_{4ik} \end{Vmatrix}; \quad V'_i = \begin{Vmatrix} V_{i1} \\ V_{i2} \\ \vdots \\ V_{in_i} \end{Vmatrix}; \quad V_{ik} = \begin{Vmatrix} v_{\alpha^A_{ik}} \\ v_{\delta^A_{ik}} \\ v_{\alpha^B_{ik}} \\ v_{\delta^B_{ik}} \end{Vmatrix}; \qquad /8/$$

$$B'_i = \begin{Vmatrix} -1 & \alpha'_{i1} & \beta'_{i1} \\ -1 & \alpha'_{i2} & \beta'_{i2} \\ \vdots & \vdots & \vdots \\ -1 & \alpha'_{in_i} & \beta'_{in_i} \end{Vmatrix}; \quad X'_i = \begin{Vmatrix} \Delta L_i \\ \Delta A_i \\ \Delta D_i \end{Vmatrix}; \quad W'_i = \begin{Vmatrix} w'_{a_{i1}} \\ w'_{a_{i2}} \\ \vdots \\ w'_{a_{in_i}} \end{Vmatrix} \qquad /8a/$$

$$(i = 1, 2, \ldots, t; \quad k = 1, 2, \ldots, n_i)$$

Coefficients $b'_{1ik}, b'_{2ik}, a'_{1ik}, a'_{2ik}, a'_{3ik}, a'_{4ik}, \alpha'_{ik}$, β'_{ik}, are calculated with the aid of corresponding formulas /3/, /4/, or /5/. Submatrixis A_i, B_i, W_z, X_z, C^*_i, and W are obtained from article [2] . System /7/ is solved by the use of formula for the adjustment from [2].

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